

α' -Corrections to extremal dyonic black holes in heterotic string theory

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ABSTRACT: We explicitly compute the entropy of an extremal dyonic black hole in heterotic string theory compactified on T^6 or $K3 \times T^2$ by taking into account all the tree level four derivative corrections to the low energy effective action. For supersymmetric black holes the result agrees with the answer obtained earlier 1) by including only the Gauss-Bonnet corrections to the effective action 2) by including all terms related to the curvature squared terms via space-time supersymmetry transformation, and 3) by using general arguments based on the assumption of AdS_3 near horizon geometry and space-time supersymmetry. For non-supersymmetric extremal black holes the result agrees with the one based on the assumption of AdS_3 near horizon geometry and space-time supersymmetry of the underlying theory.

KEYWORDS: Black Holes in String Theory, Superstrings and Heterotic Strings.

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1. Introduction

String theory at low energy describes Einstein gravity coupled to certain matter fields, together with infinite number of higher derivative corrections. Thus study of black holes in string theory involves study of black holes in higher derivative theories of gravity. While this is a complicated problem for general black holes, there are various techniques available for studying higher derivative corrections to the entropy of extremal black holes with or without supersymmetry. Nevertheless most of the analysis so far has been done by taking into account only a subset of these corrections, e.g. by including only the terms in the action proportional to Gauss-Bonnet term [1], or by including the set of all terms which are related to the curvature squared terms by supersymmetry transformation [2–9].¹ Even at the string tree level there are other four derivative terms in the action which are *a priori* equally important, and hence there is no justification for not including these terms in the analysis. Later refs. [11, 12] proved certain non-renormalization theorems establishing that for a certain class of supersymmetric black holes the results of [1, 3–9] are in fact exact. The underlying assumption behind this proof is the existence of an AdS_3 component of the near horizon geometry of the black hole solution when embedded in the full ten dimensional space-time, and supersymmetry of the resulting two dimensional theory that lives on the boundary of this AdS_3 .

Notwithstanding these non-renormalization theorems, it is important to verify the result by a direct calculation that takes into account all the higher derivative corrections in a given order. An attempt in this direction was made in [13] where the author tried to include all the tree level four derivative corrections to the action of heterotic string theory compactified on a six dimensional torus T^6 , and used this to compute correction to the entropy of an extremal dyonic black hole [14]. The apparent conclusion of this paper was that the entropy computed this way disagrees with the earlier results based on the calculations of [1–9]. If this is correct then this would also contradict the non-renormalization theorems of [11, 12]. A closer look however reveals that the analysis of [13]

¹See [10] for some discussion on the relation between these two approaches.

left out one important term, — the coupling of the gravitational Chern-Simons term to the 3-form field strength.

The purpose of this paper is to recalculate the entropy of a dyonic black hole in tree level heterotic string theory by including the complete set of tree level four derivative terms in the heterotic string effective action. We find that after the effect of gravitational Chern-Simons term is included, the resulting entropy agrees perfectly with the results of earlier analysis, in accordance with the non-renormalization theorems of [11, 12].

In carrying out our analysis we use the entropy function formalism [15] which is well suited for studying higher derivative corrections [16–19] to the entropy of extremal black holes. In the specific context of heterotic string theory in four dimensions, this formalism has been used to calculate the extremal black hole entropy in the presence of Gauss-Bonnet term [20], as well as in the presence of all terms related to the curvature squared terms via space-time supersymmetry transformaion [21]. It was also used in the analysis of [13] for computing the effect of all the four derivative terms at tree level heterotic string theory except the gravitational Chern-Simons term. In general the computation of the entropy function involves expressing the four dimensional Lagrangian density in a fully gauge and general covariant form involving only the gauge field strengths, metric, Riemann tensor, scalar fields and their covariant derivatives, and then evaluating it in a generic $SO(2,1) \times SO(3)$ invariant background reflecting the isometry of the $AdS_2 \times S^2$ near horizon geometry of an extremal black hole. For part of the four dimensional lagrangian density which comes from the dimensional reduction of a manifestly covariant six dimensional lagrangian density, the contribution to the entropy function can be related to the value of the six dimensional Lagrangian density evaluated in the corresponding six dimensional background [13]. This avoids the necessity of first dimensionally reducing the six dimensional lagrangian density to four dimensions and then evaluating its value. However this procedure fails for a part of the six dimensional lagrangian density that involves the gravitational Chern-Simons term coupled to the 3-form field strength, since this term cannot be written in a manifestly covariant form. Thus we need to first dimensionally reduce this term to four dimensions, express it in a manifestly covariant form after throwing away total derivative terms and then evaluate its value in a specific background geometry. A general procedure for dealing with dimensionally reduced Chern-Simons terms in the entropy function formalism was developed in [22]. Thus the entropy function formalism is well-suited for studying the problem at hand.

In section 2 we discuss the general strategy for dealing with the dimensional reduction of a six dimensional action that contains a gravitational Chern-Simons term in the definition of the 3-form field strength. We also discuss the strategy for computing the entropy function in such a theory. In section 3 we consider the specific example of tree level heterotic string theory compactified on T^6 or $K3 \times T^2$, analyze the complete low energy effective action up to 4-derivative terms and evaluate its contribution to the entropy function. The extremal black hole entropy, given by the value of the entropy function at its extremum, is then shown to match the results of the earlier computation of [1–9, 15, 20, 21] based on only a subset of the 4-derivative corrections to the Lagrangian density.

2. Strategy for dealing with Chern-Simons terms

We begin with the low energy effective field theory of ten dimensional heterotic string theory compactified on T^4 or $K3$. At tree level there is a consistent truncation of this theory in which we ignore all the ten dimensional gauge fields and the massless fields associated with the components of the metric and the anti-symmetric tensor fields along the compact space T^4 or $K3$. In this case the remaining massless fields consist of the string metric $G_{MN}^{(6)}$, the anti-symmetric tensor field $B_{MN}^{(6)}$ and the dilaton field $\Phi^{(6)}$ with $0 \leq M, N \leq 5$. The gauge invariant field strength associated with the anti-symmetric tensor field is given by:

$$H_{MNP}^{(6)} = \partial_M B_{NP}^{(6)} + \partial_N B_{PM}^{(6)} + \partial_P B_{MN}^{(6)} + \lambda \Omega_{MNP}^{(6)}, \quad (2.1)$$

where λ is a coefficient to be specified later and $\Omega_{MNP}^{(6)}$ denotes the gravitational Chern-Simons 3-form constructed out of the six dimensional spin connections, normalized such that

$$\begin{aligned} & \partial_Q \Omega_{MNP}^{(6)} + \text{anti-symmetrization in } P, Q, M, N \\ &= -\frac{1}{8} R_{SMN}^{(6)K} R_{K PQ}^{(6)S} + \text{anti-symmetrization in } P, Q, M, N. \end{aligned} \quad (2.2)$$

$R_{MNPQ}^{(6)}$ denotes the Riemann tensor associated with the metric $G_{MN}^{(6)}$. We shall denote the action of this theory as

$$S = \int d^6 x \sqrt{-\det G^{(6)}} \mathcal{L}^{(6)} \quad (2.3)$$

where the Lagrangian density $\mathcal{L}^{(6)}$ is a function of $G_{MN}^{(6)}$, the Riemann tensor $R_{MNPQ}^{(6)}$, $H_{MNP}^{(6)}$, $\Phi^{(6)}$ and covariant derivatives of these fields.

We shall study compactification of this theory on a two dimensional torus T^2 and study the entropy of extremal black holes in this theory. This will give rise to four abelian gauge fields from the components of the metric and the antisymmetric tensor fields along the T^2 directions. The resulting lagrangian density, besides depending on the covariant objects like the metric, Riemann tensor, gauge field strengths and their covariant derivatives, will also depend explicitly on the spin connection and the gauge fields due to the presence of the gravitational Chern-Simons term inside H_{MNP} as in (2.1) and similar gauge Chern-Simons terms which are induced during compactification [23]. Our goal is to express the effective Lagrangian density in a manifestly covariant form without involving any Chern-Simons terms so that we can apply the entropy function formalism. This will be done in two steps:

1. First at the level of the six dimensional description itself we shall introduce a new field $C_{MN}^{(6)}$ and its field strength

$$\mathcal{K}_{MNP}^{(6)} = \partial_M C_{NP}^{(6)} + \partial_N C_{PM}^{(6)} + \partial_P C_{MN}^{(6)}, \quad (2.4)$$

and consider a new Lagrangian density

$$\begin{aligned} \sqrt{-\det G^{(6)}} \tilde{\mathcal{L}}^{(6)} &\equiv \sqrt{-\det G^{(6)}} \mathcal{L}^{(6)} + \frac{1}{16\pi^2} \frac{1}{(3!)^2} \epsilon^{MNPQRS} \mathcal{K}_{MNP}^{(6)} H_{QRS}^{(6)} \\ &\quad - \frac{1}{16\pi^2} \frac{1}{(3!)^2} \lambda \epsilon^{MNPQRS} \mathcal{K}_{MNP}^{(6)} \Omega_{QRS}^{(6)} \end{aligned} \quad (2.5)$$

where we treat $H_{\text{MNP}}^{(6)}$ and $C_{\text{MN}}^{(6)}$ as independent variables. The normalization factor of $\frac{1}{16\pi^2} \frac{1}{(3!)^2}$ has been introduced for later convenience. Then we can first solve the $C_{\text{MN}}^{(6)}$ equations of motion to get the result

$$d(H^{(6)} - \lambda \Omega^{(6)}) = 0, \tag{2.6}$$

which can then be solved to get (2.1). Substituting this into (2.5) we recover the original action (2.3). On the other hand if we first eliminate $H_{\text{MNP}}^{(6)}$ by using its equation of motion, we get

$$\sqrt{-\det G^{(6)}} \tilde{\mathcal{L}}^{(6)} = \sqrt{-\det G^{(6)}} \tilde{\mathcal{L}}^{(6)'} - \frac{1}{16\pi^2} \frac{1}{(3!)^2} \lambda \epsilon^{\text{MNPQRS}} \mathcal{K}_{\text{MNP}}^{(6)} \Omega_{\text{QRS}}^{(6)} \tag{2.7}$$

where $\tilde{\mathcal{L}}^{(6)'}$ is the sum of the first two terms on the right hand side of (2.5) after elimination of $H_{\text{MNP}}^{(6)}$. This is now to be regarded as a function of the ‘dual field’ $C_{\text{MN}}^{(6)}$. $\tilde{\mathcal{L}}^{(6)'}$ depends on $C_{\text{MN}}^{(6)}$ solely through its field strength $\mathcal{K}^{(6)} \propto dC^{(6)}$ and hence has a manifestly covariant form without any Chern-Simons terms. The full Lagrangian density is still not manifestly covariant due to the presence of the Chern-Simons 3-form in the last term of (2.7).

2. We now dimensionally reduce this theory to four dimensions by introducing the fields $G_{\mu\nu}$, $C_{\mu\nu}$, Φ , \hat{G}_{mn} , \hat{C}_{mn} and $\mathcal{A}_\mu^{(i)}$ ($0 \leq \mu \leq 3$, $4 \leq m, n \leq 5$, $1 \leq i \leq 4$) via the relations

$$\begin{aligned} \hat{G}_{mn} &= G_{mn}^{(6)}, & \hat{C}_{mn} &= C_{mn}^{(6)}, \\ \hat{G}^{mn} &= (\hat{G}^{-1})^{mn}, \\ \mathcal{A}_\mu^{(m-3)} &= \frac{1}{2} \hat{G}^{mn} G_{m\mu}^{(6)}, & \mathcal{A}_\mu^{(m-1)} &= \frac{1}{2} C_{m\mu}^{(6)} - \hat{C}_{mn} \mathcal{A}_\mu^{(n-3)}, \\ G_{\mu\nu} &= G_{\mu\nu}^{(6)} - \hat{G}^{mn} G_{m\mu}^{(6)} G_{n\nu}^{(6)}, \\ C_{\mu\nu} &= C_{\mu\nu}^{(6)} - 4\hat{C}_{mn} \mathcal{A}_\mu^{(m-3)} \mathcal{A}_\nu^{(n-3)} - 2(\mathcal{A}_\mu^{(m-3)} \mathcal{A}_\nu^{(m-1)} - \mathcal{A}_\nu^{(m-3)} \mathcal{A}_\mu^{(m-1)}), \\ \Phi &= \Phi^{(6)} - \frac{1}{2} \ln V_{\mathcal{M}}, \end{aligned} \tag{2.8}$$

where x^4 and x^5 are the coordinates labelling the torus and $V_{\mathcal{M}}$ is the volume of T^2 measured in the string metric. We shall normalize x^4 and x^5 so that they have coordinate radius 1. Then

$$V_{\mathcal{M}} = 4\pi^2 \sqrt{\det \hat{G}}. \tag{2.9}$$

The gauge invariant field strengths associated with $\mathcal{A}_\mu^{(i)}$ and $C_{\mu\nu}$ are

$$\mathcal{F}_{\mu\nu}^{(i)} = \partial_\mu \mathcal{A}_\nu^{(i)} - \partial_\nu \mathcal{A}_\mu^{(i)}, \quad 1 \leq i, j \leq 4, \tag{2.10}$$

$$\mathcal{K}_{\mu\nu\rho} = (\partial_\mu C_{\nu\rho} + 2\mathcal{A}_\mu^{(i)} L_{ij} \mathcal{F}_{\nu\rho}^{(j)}) + \text{cyclic permutations of } \mu, \nu, \rho, \tag{2.11}$$

where

$$L = \begin{pmatrix} 0 & I_2 \\ I_2 & 0 \end{pmatrix}, \tag{2.12}$$

I_2 being 2×2 identity matrix. In this case the Lagrangian density, obtained by dimensional reduction of the right hand side of (2.7) has the form

$$\sqrt{-\det G} \tilde{\mathcal{L}} = \sqrt{-\det G} \tilde{\mathcal{L}}' + \sqrt{-\det G} \tilde{\mathcal{L}}'', \quad (2.13)$$

where

$$\sqrt{-\det G} \tilde{\mathcal{L}}' = \int dx^4 dx^5 \sqrt{-\det G^{(6)}} \tilde{\mathcal{L}}^{(6)'}, \quad (2.14)$$

$$\sqrt{-\det G} \tilde{\mathcal{L}}'' = -\frac{1}{16\pi^2} \frac{1}{(3!)^2} \lambda \int dx^4 dx^5 \epsilon^{\text{MNPQRS}} \mathcal{K}_{\text{MNP}}^{(6)} \Omega_{\text{QRS}}^{(6)} + \text{total derivative terms.} \quad (2.15)$$

$\tilde{\mathcal{L}}'$ is a function of the field strength $\mathcal{K}_{\mu\nu\rho}$ and other covariant objects. We shall explicitly demonstrate that $\tilde{\mathcal{L}}''$ is also a function of the field strengths and other covariant objects after we remove certain total derivative terms. However due to the presence of explicit gauge fields in the expression for $\mathcal{K}_{\mu\nu\rho}$ this form of the Lagrangian density is not suitable for applying the entropy function method. For this we dualize this action further by replacing the Lagrangian density $\sqrt{-\det G} \tilde{\mathcal{L}}$ by

$$\sqrt{-\det G} \tilde{\mathcal{L}} + \epsilon^{\mu\nu\rho\sigma} \mathcal{K}_{\mu\nu\rho} \partial_\sigma b + 3b \epsilon^{\mu\nu\rho\sigma} \mathcal{F}_{\sigma\mu}^{(i)} L_{ij} \mathcal{F}_{\nu\rho}^{(j)}, \quad (2.16)$$

and treating $\mathcal{K}_{\mu\nu\rho}$ and the new scalar field b as independent variables. If we choose to first use the equation of motion of the b field then we get

$$\epsilon^{\mu\nu\rho\sigma} \partial_\sigma \left(\mathcal{K}_{\mu\nu\rho} - 6\mathcal{A}_\mu^{(i)} L_{ij} \mathcal{F}_{\nu\rho}^{(j)} \right) = 0, \quad (2.17)$$

which has as its solution the form (2.11) for some $C_{\mu\nu}$. Substituting this into (2.16) we recover the original action (2.13) up to total derivative terms. On the other hand if we first eliminate $\mathcal{K}_{\mu\nu\rho}$ from (2.16) by its equation of motion we shall get a Lagrangian density of the form:

$$\sqrt{-\det G} \hat{\mathcal{L}} = \sqrt{-\det G} \hat{\mathcal{L}}' + 3b \epsilon^{\mu\nu\rho\sigma} \mathcal{F}_{\sigma\mu}^{(i)} L_{ij} \mathcal{F}_{\nu\rho}^{(j)}, \quad (2.18)$$

where $\hat{\mathcal{L}}'$, obtained by substituting the solution for $\mathcal{K}_{\mu\nu\rho}$ in the first two terms in (2.16), has a manifestly covariant expression in terms of $\partial_\sigma b$ and other covariant objects. This way we arrive at a manifestly covariant form of the Lagrangian density for which we can apply the entropy function formalism.

Let us now say a few words about the evaluation of the entropy function \mathcal{E} . For this we need to consider a general $AdS_2 \times S^2$ near horizon geometry with all other background field configurations consistent with the symmetries of $AdS_2 \times S^2$ and define

$$\mathcal{E} = 2\pi \left(\sum_{i=1}^4 \tilde{q}_i \tilde{e}_i - \int d\theta d\phi \sqrt{-\det G} \hat{\mathcal{L}} \right), \quad (2.19)$$

evaluated in this background. Here \tilde{q}_i denotes the electric charge associated with the gauge field $\mathcal{A}_\mu^{(i)}$ and \tilde{e}_i denotes the value of the radial electric field $\mathcal{F}_{rt}^{(i)}$. Since (2.18) is obtained from (2.16) after elimination of the variables $\mathcal{K}_{\mu\nu\rho}$, and since the right hand side of (2.16)

is manifestly covariant when $\mathcal{K}_{\mu\nu\rho}$ is interpreted as an auxiliary field, we can replace the $\sqrt{-\det G} \widehat{\mathcal{L}}$ on the right hand side of (2.19) by the right hand side of (2.16). Since both $\partial_\sigma b$ and $\mathcal{K}_{\mu\nu\rho}$ vanish in an $AdS_2 \times S^2$ geometry due to the absence of $SO(2,1) \times SO(3)$ invariant 1- and 3-forms, we can set them to zero in (2.16) during the computation of the entropy function. Thus we have

$$\mathcal{E} = 2\pi \left(\sum_{i=1}^4 \tilde{q}_i \tilde{e}_i - \int d\theta d\phi \sqrt{-\det G} \tilde{\mathcal{L}} - 3 \int d\theta d\phi b \epsilon^{\mu\nu\rho\sigma} \mathcal{F}_{\sigma\mu}^{(i)} L_{ij} \mathcal{F}_{\nu\rho}^{(j)} \right). \quad (2.20)$$

Using eqs. (2.13), (2.14) we can express this as

$$\begin{aligned} \mathcal{E} = 2\pi & \left(\sum_{i=1}^4 \tilde{q}_i \tilde{e}_i - \int d\theta d\phi dx^4 dx^5 \sqrt{-\det G^{(6)}} \tilde{\mathcal{L}}^{(6)'} - \int d\theta d\phi \sqrt{-\det G} \tilde{\mathcal{L}}'' \right. \\ & \left. - 3 \int d\theta d\phi b \epsilon^{\mu\nu\rho\sigma} \mathcal{F}_{\sigma\mu}^{(i)} L_{ij} \mathcal{F}_{\nu\rho}^{(j)} \right). \end{aligned} \quad (2.21)$$

Finally, using (2.5), (2.7) we can express this as

$$\begin{aligned} \mathcal{E} = 2\pi & \left[\sum_{i=1}^4 \tilde{q}_i \tilde{e}_i - \int d\theta d\phi dx^4 dx^5 \left(\sqrt{-\det G^{(6)}} \mathcal{L}^{(6)} + \frac{1}{16\pi^2} \frac{1}{(3!)^2} \epsilon^{\text{MNPQRS}} \mathcal{K}_{\text{MNP}}^{(6)} H_{\text{QRS}}^{(6)} \right) \right. \\ & \left. - \int d\theta d\phi \sqrt{-\det G} \tilde{\mathcal{L}}'' - 3 \int d\theta d\phi b \epsilon^{\mu\nu\rho\sigma} \mathcal{F}_{\sigma\mu}^{(i)} L_{ij} \mathcal{F}_{\nu\rho}^{(j)} \right], \end{aligned} \quad (2.22)$$

where $H_{\text{MNP}}^{(6)}$ needs to be interpreted as an elementary auxiliary field which has to be eliminated by its equation of motion. The terms in the first line of (2.22) can be evaluated by regarding the background as a six dimensional configuration. Thus we do not need to explicitly find the dimensional reduction of this term. For the contribution from the $\tilde{\mathcal{L}}''$ term however we cannot directly evaluate the six dimensional form proportional to $\int dx^4 dx^5 \epsilon^{\text{MNPQRS}} \mathcal{K}_{\text{MNP}}^{(6)} \Omega_{\text{QRS}}^{(6)}$ due to the presence of the total derivative terms in (2.15). We need to first find its dimensional reduction to four dimensions and then use this to calculate the entropy function.

So far we have not made any approximation. What we are interested in however is an approximation scheme where we take into account higher derivative corrections to the effective action in a power series expansion. In particular we shall be interested in the correction due to the four derivative terms in the action. For this let us split the original Lagrangian density $\mathcal{L}^{(6)}$ as

$$\mathcal{L}^{(6)} = \mathcal{L}_0^{(6)} + \mathcal{L}_1^{(6)}, \quad (2.23)$$

where $\mathcal{L}_0^{(6)}$ denotes the supergravity Lagrangian density and $\mathcal{L}_1^{(6)}$ denotes four derivative corrections. The entropy function obtained from this Lagrangian density has the form:

$$\mathcal{E} = \mathcal{E}_0 + \mathcal{E}_1, \quad (2.24)$$

with \mathcal{E}_0 and \mathcal{E}_1 reflecting the contribution from the two and four derivative terms respectively:

$$\mathcal{E}_0 = 2\pi \left(\sum_{i=1}^4 \tilde{q}_i \tilde{e}_i - \int d\theta d\phi dx^4 dx^5 \left(\sqrt{-\det G^{(6)}} \mathcal{L}_0^{(6)} + \frac{1}{16\pi^2} \frac{1}{(3!)^2} \epsilon^{\text{MNPQRS}} \mathcal{K}_{\text{MNP}}^{(6)} H_{\text{QRS}}^{(6)} \right) - 3 \int d\theta d\phi b \epsilon^{\mu\nu\rho\sigma} \mathcal{F}_{\sigma\mu}^{(i)} L_{ij} \mathcal{F}_{\nu\rho}^{(j)} \right), \quad (2.25)$$

$$\mathcal{E}_1 = 2\pi \left(- \int d\theta d\phi dx^4 dx^5 \sqrt{-\det G^{(6)}} \mathcal{L}_1^{(6)} - \int d\theta d\phi \sqrt{-\det G} \tilde{\mathcal{L}}'' \right). \quad (2.26)$$

Since the entropy is given by the value of \mathcal{E} at its *extremum*, a first order error in the determination of the near horizon background will give a second order error in the value of the entropy. Thus we can find the near horizon background, including the auxiliary field $H_{\text{MNP}}^{(6)}$, by extremizing \mathcal{E}_0 and then evaluate $\mathcal{E}_0 + \mathcal{E}_1$ in this background. This gives the value of the entropy correctly up to first order.

3. Computation of the entropy

We shall now compute the entropy function for heterotic string theory compactified on T^6 or $K3 \times T^2$ following the strategy outlined in the previous section. We begin with the computation of \mathcal{E}_0 . In the $\alpha' = 16$ unit that we shall be using in order to facilitate comparison with previous results (*e.g* that of [20]), the relevant bosonic part of the Lagrangian density $\mathcal{L}_0^{(6)}$, describing heterotic string theory compactified on T^4 or $K3$, can be expressed as

$$\mathcal{L}_0^{(6)} = \frac{1}{32\pi} e^{-2\Phi^{(6)}} \left[R^{(6)} + 4\partial_M \Phi^{(6)} \partial^M \Phi^{(6)} - \frac{1}{12} H_{\text{MNP}}^{(6)} H^{(6)\text{MNP}} \right], \quad (3.1)$$

where all the indices are raised and lowered by the six dimensional string metric $G_{\text{MN}}^{(6)}$. In writing down this expression we have set to zero all the ten dimensional gauge fields as well as the gauge and moduli fields associated with the compact space T^4 or $K3$. This is a consistent truncation of the theory. Thus at this order $H_{\text{MNP}}^{(6)}$, obtained by extremizing \mathcal{E}_0 given in (2.25), is given by

$$H^{(6)\text{MNP}} = -\frac{1}{3!} \frac{2}{\pi} \left(\sqrt{-\det G^{(6)}} \right)^{-1} e^{2\Phi^{(6)}} \epsilon^{\text{MNPQRS}} \mathcal{K}_{\text{QRS}}^{(6)}. \quad (3.2)$$

As discussed after eq. (2.26), we can continue to use this result even at next order if we want to calculate the correction to the black hole entropy up to four derivative terms.

After dimensional reduction given in (2.8) we get a four dimensional theory. We consider an extremal black hole solution in this theory with near horizon configuration:

$$\begin{aligned} ds^2 &\equiv G_{\mu\nu} dx^\mu dx^\nu = v_1 \left(-r^2 dt^2 + \frac{dr^2}{r^2} \right) + v_2 (d\theta^2 + \sin^2 \theta d\phi^2), \\ \hat{G} &= \begin{pmatrix} u_1^2 & 0 \\ 0 & u_2^2 \end{pmatrix}, \quad \hat{C} = 0, \quad e^{-2\Phi} = u_S, \quad b = 0, \\ \mathcal{F}_{rt}^{(1)} &= \tilde{e}_1, \quad \mathcal{F}_{rt}^{(3)} = \tilde{e}_3, \quad \mathcal{F}_{\theta\phi}^{(2)} = \frac{\tilde{p}_2}{4\pi} \sin \theta, \quad \mathcal{F}_{\theta\phi}^{(4)} = \frac{\tilde{p}_4}{4\pi} \sin \theta. \end{aligned} \quad (3.3)$$

We have set the off-diagonal components of \widehat{G} , \widehat{C} , the scalar field b and some components of the electromagnetic field strengths to zero by requiring the field configuration to be invariant under $x^5 \rightarrow -x^5$, $x^i \rightarrow -x^i$ for $1 \leq i \leq 3$. Using (2.8) we see that this corresponds to the following six dimensional field configuration:

$$\begin{aligned} ds_6^2 &\equiv G_{MN}^{(6)} dx^M dx^N = ds^2 + u_1^2 (dx^4 + 2\tilde{e}_1 r dt)^2 + u_2^2 \left(dx^5 - \frac{\tilde{p}_2}{2\pi} \cos \theta d\phi \right)^2, \\ C_{4t}^{(6)} &= 2\tilde{e}_3 r, \quad C_{5\phi}^{(6)} = -\frac{\tilde{p}_4}{2\pi} \cos \theta, \\ e^{-2\Phi^{(6)}} &= \frac{u_S}{4\pi^2 u_1 u_2}, \end{aligned} \quad (3.4)$$

which gives

$$\mathcal{K}_{rt4}^{(6)} = -2\tilde{e}_3, \quad \mathcal{K}_{\theta\phi 5}^{(6)} = -\frac{\tilde{p}_4}{2\pi} \sin \theta. \quad (3.5)$$

We shall use the convention

$$\epsilon^{tr\theta\phi 45} = 1. \quad (3.6)$$

Eq. (3.2) then gives

$$\begin{aligned} H^{(6)rt4} &= \frac{2}{\pi} (\sqrt{-\det G^{(6)}})^{-1} e^{2\Phi^{(6)}} \mathcal{K}_{\theta\phi 5}^{(6)} = -\frac{4}{v_1 v_2 u_S} \tilde{p}_4, \\ H^{(6)\theta\phi 5} &= -\frac{2}{\pi} (\sqrt{-\det G^{(6)}})^{-1} e^{2\Phi^{(6)}} \mathcal{K}_{rt4}^{(6)} = \frac{16\pi}{v_1 v_2 u_S \sin \theta} \tilde{e}_3. \end{aligned} \quad (3.7)$$

For this specific configuration (2.25) gives the leading order entropy function to be

$$\mathcal{E}_0 = 2\pi \left[\tilde{e}_1 \tilde{q}_1 + \tilde{e}_3 \tilde{q}_3 - \frac{1}{8} v_1 v_2 u_S \left(-\frac{2}{v_1} + \frac{2}{v_2} + \frac{2u_1^2 \tilde{e}_1^2}{v_1^2} + \frac{128\pi^2 u_2^2 \tilde{e}_3^2}{v_1^2 u_S^2} - \frac{u_2^2 \tilde{p}_2^2}{8\pi^2 v_2^2} - \frac{8u_1^2 \tilde{p}_4^2}{v_2^2 u_S^2} \right) \right]. \quad (3.8)$$

Extremizing this with respect to \tilde{e}_1 and \tilde{e}_3 and substituting their values back in (3.8) we get

$$\tilde{e}_1 = \frac{2v_1 \tilde{q}_1}{v_2 u_S u_1^2}, \quad \tilde{e}_3 = \frac{v_1 u_S \tilde{q}_3}{32\pi^2 v_2 u_2^2}. \quad (3.9)$$

and

$$\mathcal{E}_0 = \frac{\pi}{4} v_1 v_2 u_S \left[\frac{2}{v_1} - \frac{2}{v_2} + \frac{8\tilde{q}_1^2}{v_2^2 u_S^2 u_1^2} + \frac{\tilde{q}_3^2}{8\pi^2 v_2^2 u_2^2} + \frac{u_2^2 \tilde{p}_2^2}{8\pi^2 v_2^2} + \frac{8u_1^2 \tilde{p}_4^2}{v_2^2 u_S^2} \right]. \quad (3.10)$$

In this form the entropy function cannot be directly compared with the earlier results of [20], since we have defined the gauge fields $\mathcal{A}_\mu^{(3)}$ and $\mathcal{A}_\mu^{(4)}$ via dimensional reduction of the fields $C_{MN}^{(6)}$ whereas the gauge fields $A_\mu^{(3)}$ and $A_\mu^{(4)}$ of ref. [20] would come from the dimensional reduction of the anti-symmetric tensor field $B_{MN}^{(6)}$ which are dual to the fields $C_{MN}^{(6)}$. We can find the relation between the charges $(\tilde{p}_i, \tilde{q}_i)$ and the charges (p_i, q_i) of [20] by comparing the expressions for $H^{(6)MNP}$ given in (3.7) with the corresponding expressions in [20], and then using the relation between the near horizon fields and charges in both description. This gives

$$q_1 = \tilde{q}_1, \quad p_2 = \tilde{p}_2, \quad q_3 = -\tilde{p}_4, \quad p_4 = -\tilde{q}_3. \quad (3.11)$$

(3.10) may now be rewritten as

$$\mathcal{E}_0 = \frac{\pi}{4} v_1 v_2 u_S \left[\frac{2}{v_1} - \frac{2}{v_2} + \frac{8q_1^2}{v_2^2 u_S^2 u_1^2} + \frac{p_4^2}{8\pi^2 v_2^2 u_2^2} + \frac{u_2^2 p_2^2}{8\pi^2 v_2^2} + \frac{8u_1^2 q_3^2}{v_2^2 u_S^2} \right]. \quad (3.12)$$

This agrees with the entropy function computed in [20].

The relations (3.11) between the two sets of charges depend on the precise normalization of the dual field $\mathcal{K}_{\text{MNP}}^{(6)}$ and the definition of the four dimensional gauge fields in terms of the six dimensional fields, but not on the details of the Lagrangian density $\mathcal{L}^{(6)}$. Thus (3.11) continues to hold even after inclusion of higher derivative corrections to the action. In order to facilitate comparison with the known results we shall express all answers in terms of the charges q_1 , q_3 , p_2 and p_4 from now on. Physically these charges represent n unit of momentum and w unit of winding charge along x^4 and N' unit of Kaluza-Klein monopole and W' unit of H-monopole charge associated with the circle along x^5 , with [20]

$$q_1 = \frac{1}{2} n, \quad q_3 = \frac{1}{2} w, \quad p_2 = 4\pi N', \quad p_4 = 4\pi W'. \quad (3.13)$$

Extremizing (3.12) with respect to v_1 , v_2 , u_1 , u_2 and u_S and using (3.9) we get

$$\begin{aligned} v_1 = v_2 &= \frac{1}{4\pi^2} |p_2 p_4|, & u_S &= 8\pi \sqrt{\left| \frac{q_1 q_3}{p_2 p_4} \right|}, & u_1 &= \sqrt{\left| \frac{q_1}{q_3} \right|}, & u_2 &= \sqrt{\left| \frac{p_4}{p_2} \right|} \\ \tilde{e}_1 &= \frac{1}{4\pi q_1} \sqrt{|p_2 p_4 q_1 q_3|}, & \tilde{e}_3 &= -\frac{1}{4\pi p_4} \sqrt{|p_2 p_4 q_1 q_3|}. \end{aligned} \quad (3.14)$$

Substituting this back into (3.12) we get the leading order contribution to the black hole entropy:

$$\mathcal{E}_0 = \sqrt{|p_2 p_4 q_1 q_3|} = 2\pi \sqrt{|nwN'W'|}. \quad (3.15)$$

We now turn to the evaluation of \mathcal{E}_1 . We shall divide the contribution into two parts:

$$\mathcal{E}_1 = \mathcal{E}'_1 + \mathcal{E}''_1, \quad (3.16)$$

where

$$\mathcal{E}'_1 = -2\pi \int d\theta d\phi dx^4 dx^5 \sqrt{-\det G^{(6)}} \mathcal{L}_1^{(6)} \quad (3.17)$$

and

$$\mathcal{E}''_1 = - \int d\theta d\phi \sqrt{-\det G} \tilde{\mathcal{L}}''. \quad (3.18)$$

First let us compute \mathcal{E}'_1 . For this we need the expression for the four derivative corrections to the heterotic string effective action at the string tree level. This is given by [24, 25]

$$\begin{aligned} \mathcal{L}_1^{(6)} &= \frac{1}{16\pi} e^{-2\Phi^{(6)}} \left[R_{\text{KLMN}}^{(6)} R^{(6)KLMN} - \frac{1}{2} R_{\text{KLMN}}^{(6)} H_P^{(6)KL} H^{(6)PMN} \right. \\ &\quad \left. - \frac{1}{8} H_K^{(6)MN} H_{\text{LMN}}^{(6)} H^{(6)KPQ} H_{\text{PQ}}^{(6)L} + \frac{1}{24} H_{\text{KLM}}^{(6)} H_{\text{PQ}}^{(6)K} H_R^{(6)LP} H^{(6)RMQ} \right]. \end{aligned} \quad (3.19)$$

Using (3.4)-(3.7) and (3.17) we get [13]

$$\mathcal{E}'_1 = -4\pi v_1 v_2 u_S \left[\frac{1}{2v_1^2} + \frac{1}{2v_2^2} - \frac{3\tilde{e}_1^2 u_1^2}{v_1^3} - \frac{3\tilde{p}_2^2 u_2^2}{16v_2^3 \pi^2} + \frac{11u_1^4 \tilde{e}_1^4}{2v_1^4} + \frac{11\tilde{p}_2^4 u_2^4}{512v_2^4 \pi^4} \right. \\ \left. - \frac{4u_1^2 \tilde{p}_4^2}{v_1 v_2^2 u_S^2} - \frac{64\pi^2 u_2^2 \tilde{e}_3^2}{v_1^2 v_2 u_S^2} + \frac{4u_1^4 \tilde{p}_4^2 \tilde{e}_1^2}{v_1^2 v_2^2 u_S^2} + \frac{4u_2^4 \tilde{e}_3^2 \tilde{p}_2^2}{v_1^2 v_2^2 u_S^2} \right. \\ \left. - \frac{40u_1^4 \tilde{p}_4^4}{v_2^4 u_S^4} - \frac{10240 \pi^4 u_2^4 \tilde{e}_3^4}{v_1^4 u_S^4} \right]. \quad (3.20)$$

As discussed below eq. (2.26), in computing the black hole entropy we can substitute the solution given in (3.14), obtained by extremizing \mathcal{E}_0 , into the expression for \mathcal{E}_1 . This gives the contribution to the black hole entropy from \mathcal{E}'_1 to be [13]

$$\mathcal{E}'_1 = 16\pi^2 \sqrt{\left| \frac{q_1 q_3}{p_2 p_4} \right|}. \quad (3.21)$$

Let us now turn to the computation of \mathcal{E}''_1 . This would require first dimensionally reducing the Chern-Simons term to construct a covariant four dimensional Lagrangian density via eq. (2.15), and then computing its contribution to the entropy function via eq. (3.18). This analysis can be simplified by regarding the sphere labelled by θ, ϕ also as a compact space and considering dimensional reduction of (2.15) all the way to two dimensions spanned by the coordinates r and t . The resulting two dimensional Lagrangian density has the form

$$\sqrt{-\det G^{(2)}} \tilde{\mathcal{L}}^{(2)''} = -\frac{1}{16\pi^2} \frac{1}{(3!)^2} \lambda \int dx^4 dx^5 d\theta d\phi \epsilon^{MNPQRS} \mathcal{K}_{MNP}^{(6)} \Omega_{QRS}^{(6)} \\ + \text{total derivative terms}, \quad (3.22)$$

where the total derivative terms need to be chosen such that the $\tilde{\mathcal{L}}^{(2)''}$ is manifestly covariant. The contribution \mathcal{E}''_1 to the entropy function is then given by

$$\mathcal{E}''_1 = -2\pi \sqrt{-\det G^{(2)}} \tilde{\mathcal{L}}^{(2)''}, \quad (3.23)$$

evaluated in the near horizon background of the black hole.

We can carry out the dimensional reduction from six to two dimensions in two stages. First of all we note that the six dimensional field configuration given in (3.4) has the structure of a product of two three dimensional spaces, the first one labelled by (θ, ϕ, x^5) and the second one labelled by (t, r, x^4) . Thus we can make a consistent truncation where we consider only those field configurations which respect this product structure. In this case (3.22) simplifies to

$$\sqrt{-\det G^{(2)}} \tilde{\mathcal{L}}^{(2)''} = -\frac{1}{16\pi^2} \frac{1}{(3!)^2} \lambda \int dx^4 dx^5 d\theta d\phi \epsilon^{\tilde{m}\tilde{n}\tilde{p}} \epsilon^{\check{\alpha}\check{\beta}\check{\gamma}} (\mathcal{K}_{\tilde{m}\tilde{n}\tilde{p}}^{(6)} \Omega_{\check{\alpha}\check{\beta}\check{\gamma}}^{(6)} - \Omega_{\tilde{m}\tilde{n}\tilde{p}}^{(6)} \mathcal{K}_{\check{\alpha}\check{\beta}\check{\gamma}}^{(6)}) \quad (3.24)$$

where the indices $\tilde{m}, \tilde{n}, \tilde{p}$ run over (θ, ϕ, x^5) and the indices $\check{\alpha}, \check{\beta}, \check{\gamma}$ run over (t, r, x^4) . We have chosen the following convention for the three dimensional ϵ tensors:

$$\epsilon^{tr4} = 1, \quad \epsilon^{\theta\phi5} = 1. \quad (3.25)$$

Let us now label the components of the six dimensional metric as

$$G_{\check{m}\check{n}}^{(6)} dx^{\check{m}} dx^{\check{n}} = G_{55}^{(6)} \left(h_{mn} dx^m dx^n + (dx^5 + 2\mathcal{A}_m^{(2)} dx^m)^2 \right) \quad (3.26)$$

and

$$G_{\check{\alpha}\check{\beta}}^{(6)} dx^{\check{\alpha}} dx^{\check{\beta}} = G_{44}^{(6)} \left(g_{\alpha\beta} dx^\alpha dx^\beta + (dx^4 + 2\mathcal{A}_\alpha^{(1)} dx^\alpha)^2 \right) \quad (3.27)$$

where the indices m, n run over (θ, ϕ) and the indices α, β run over (t, r) . Then it follows from the analysis of [26, 22] that

$$\int dx^5 d\theta d\phi \epsilon^{\check{m}\check{n}\check{p}} \Omega_{\check{m}\check{n}\check{p}}^{(6)} = \pi \int d\theta d\phi \epsilon^{mn} \left[R_h \mathcal{F}_{mn}^{(2)} + 4 h^{m'p'} h^{q'q} \mathcal{F}_{mm'}^{(2)} \mathcal{F}_{p'q'}^{(2)} \mathcal{F}_{qn}^{(2)} \right] \quad (3.28)$$

and

$$\int dx^4 \epsilon^{\check{\alpha}\check{\beta}\check{\gamma}} \Omega_{\check{\alpha}\check{\beta}\check{\gamma}}^{(6)} = \pi \epsilon^{\alpha\beta} \left[R_g \mathcal{F}_{\alpha\beta}^{(1)} + 4 g^{\alpha'\gamma'} g^{\delta'\delta} \mathcal{F}_{\alpha\alpha'}^{(1)} \mathcal{F}_{\gamma'\delta'}^{(1)} \mathcal{F}_{\delta\beta}^{(1)} \right] + \text{total derivative terms} \quad (3.29)$$

where R_h and R_g denotes the scalar curvature associated with the metrics h_{mn} and $g_{\alpha\beta}$ respectively. Our convention for the two dimensional ϵ tensor is

$$\epsilon^{tr} = 1, \quad \epsilon^{\theta\phi} = 1. \quad (3.30)$$

Thus we get

$$\begin{aligned} \sqrt{-\det G^{(2)}} \tilde{\mathcal{L}}^{(2)''} = & -\frac{1}{16\pi} \frac{1}{(3!)^2} \lambda \left[6\pi \left(\int d\theta d\phi \epsilon^{mn} \mathcal{K}_{5mn}^{(6)} \right) \epsilon^{\alpha\beta} \left[R_g \mathcal{F}_{\alpha\beta}^{(1)} + 4 g^{\alpha'\gamma'} g^{\delta'\delta} \mathcal{F}_{\alpha\alpha'}^{(1)} \mathcal{F}_{\gamma'\delta'}^{(1)} \mathcal{F}_{\delta\beta}^{(1)} \right] \right. \\ & \left. - 6\pi \left(\int d\theta d\phi \epsilon^{mn} \left[R_h \mathcal{F}_{mn}^{(2)} + 4 h^{m'p'} h^{q'q} \mathcal{F}_{mm'}^{(2)} \mathcal{F}_{p'q'}^{(2)} \mathcal{F}_{qn}^{(2)} \right] \right) \epsilon^{\alpha\beta} \mathcal{K}_{4\alpha\beta}^{(6)} \right]. \quad (3.31) \end{aligned}$$

Since the lagrangian density now has manifest covariance, we can apply the entropy function formalism. This requires evaluating the right hand side of (3.31) for the six dimensional background given in (3.4). Noting that for this configuration

$$h_{mn} dx^m dx^n = v_2 u_2^{-2} (d\theta^2 + \sin^2 \theta d\phi^2), \quad g_{\alpha\beta} dx^\alpha dx^\beta = v_1 u_1^{-2} (-r^2 dt^2 + dr^2/r^2), \quad (3.32)$$

we get

$$\sqrt{-\det G^{(2)}} \tilde{\mathcal{L}}^{(2)''} = \frac{2\lambda\pi}{3} \left[\frac{\tilde{p}_4}{4\pi} \left(\frac{u_1^2}{v_1} \tilde{e}_1 - 2 \frac{u_1^4}{v_1^2} \tilde{e}_1^3 \right) + \tilde{e}_3 \left(\frac{u_2^2}{v_2} \frac{\tilde{p}_2}{4\pi} - 2 \frac{u_2^4}{v_2^2} \left(\frac{\tilde{p}_2}{4\pi} \right)^3 \right) \right]. \quad (3.33)$$

Evaluating this for the solution given in (3.14) we get

$$\mathcal{E}_1'' = -2\pi \sqrt{-\det G^{(2)}} \tilde{\mathcal{L}}^{(2)''} = \frac{1}{6} \lambda \pi^2 \left(\frac{q_1 q_3}{\sqrt{|p_2 p_4 q_1 q_3|}} + \frac{\sqrt{|p_2 p_4 q_1 q_3|}}{p_2 p_4} \right) \quad (3.34)$$

For definiteness we shall now consider the range of values

$$p_2 > 0, \quad p_4 > 0, \quad q_3 > 0. \quad (3.35)$$

In this case the full black hole entropy, given by the value of the entropy function at its extremum, becomes

$$\mathcal{E} = \mathcal{E}_0 + \mathcal{E}'_1 + \mathcal{E}''_1 = \sqrt{|p_2 p_4 q_1 q_3|} \left[1 + \frac{\pi^2}{p_2 p_4} \left\{ 16 + \frac{1}{6} \lambda \left(1 + \frac{q_1}{|q_1|} \right) \right\} \right]. \quad (3.36)$$

Let us now turn to the determination of the parameter λ . If we define

$$a = 8\pi C_{45}^{(6)}, \quad (3.37)$$

then after elimination of $H_{\text{MNP}}^{(6)}$ using (3.2) and dimensional reduction to four dimensions, the action contains the terms:

$$\frac{1}{32\pi} \int d^4x \left[-\frac{1}{2} \sqrt{-\det G} e^{2\Phi} G^{\mu\nu} \partial_\mu a \partial_\nu a + \frac{\lambda}{48} a \epsilon^{\mu\nu\rho\sigma} R^c{}_{d\mu\nu} R^d{}_{c\rho\sigma} + \dots \right]. \quad (3.38)$$

a plays the role of the axion field. Comparing this with the standard action for tree level heterotic string theory (see e.g. [21]) compactified down to four dimensions, we get

$$\lambda = 48. \quad (3.39)$$

Eq. (3.36) now gives

$$\begin{aligned} \mathcal{E} &= \sqrt{|p_2 p_4 q_1 q_3|} \left[1 + 32 \frac{\pi^2}{p_2 p_4} \right] = 2\pi \sqrt{|nwN'W'|} \left[1 + \frac{2}{N'W'} \right] && \text{for } q_1 > 0, \\ &= \sqrt{|p_2 p_4 q_1 q_3|} \left[1 + 16 \frac{\pi^2}{p_2 p_4} \right] = 2\pi \sqrt{|nwN'W'|} \left[1 + \frac{1}{N'W'} \right] && \text{for } q_1 < 0. \end{aligned} \quad (3.40)$$

For $q_1 > 0$ the black hole is supersymmetric. The result for the entropy agrees with the result obtained by 1) including only the Gauss-Bonnet term in the four dimensional effective action [1, 20], 2) including a fully supersymmetrized version of the curvature squared correction in the four dimensional effective action [3–9] and 3) the argument based on the existence of an AdS_3 component of the near horizon geometry and supersymmetry of the associated boundary theory [11, 12]. Since the last result makes use of supersymmetry to relate the gauge anomaly to the trace anomaly in the boundary theory, our result provides an indirect evidence that the bosonic effective action given in (3.19) can be consistently supersymmetrized to this order in α' .

We also see from (3.36) that

$$\mathcal{E}_{q_1>0} - \mathcal{E}_{q_1<0} = 16 \sqrt{|p_2 p_4 q_1 q_3|} \frac{\pi^2}{p_2 p_4}. \quad (3.41)$$

This agrees with the result derived under the assumption that the subspace spanned by the coordinates x^4 , t and r form a locally AdS_3 space time near the horizon [11, 12, 21, 22].

Finally we note that for heterotic string theory compactified on T^6 or more general $\mathcal{N} = 4$ supersymmetric string compactification, the statistical entropy of some of these black holes can be computed exactly by representing them as a configuration of D-branes and Kaluza-Klein monopoles in the dual type IIA string theory [27–33]. The approximation

used here by restricting to tree level heterotic string theory will be a valid approximation if the near horizon value of the string coupling constant is small. (3.14) shows that this requires the electric charges q_1, q_3 to be large compared to the magnetic charges p_2, p_4 . Within this approximation the result for the statistical entropy is known to agree with the black hole entropy computed using the Gauss-Bonnet term [34, 30, 31, 33]. Hence this also agrees with the results found here by including the complete set of higher derivative terms.

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